

Congested Clique Counting for Local Gibbs Distributions

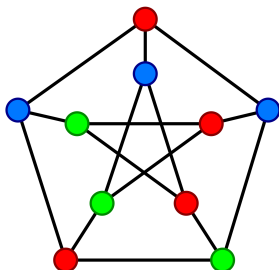
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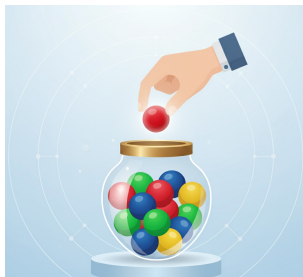
Background Information

- Goal is to estimate the size of a large set
- Estimate N by $\hat{N}$ such that $(1 - \epsilon)N \leq \hat{N} \leq (1 + \epsilon)N$
- For many problems can be done with $O(\frac{n}{\epsilon^2})$ samples
Stefankovic, Vempala, Vigoda (FOCS '07)
- Improved to allow $O(\frac{n}{\epsilon^2})$ samples taken in parallel
Liu, Yin, Zhang (arxiv '25)
- Focus on graph colorings



Sampling Colorings

- Markov chain from Feng, Sun, and Yin (PODC '17)
- Start from arbitrary coloring X
- Each round proceeds as follows
 - Each vertex v randomly proposes a color, σ_v
 - Each edge $\{u, v\} \in E$ passes iff
 - $\sigma_v \neq \sigma_u$
 - $\sigma_v \neq X_u$
 - $\sigma_u \neq X_v$
 - If every edge of v passes, $X_v = \sigma_v$
- Repeat $O(\log n)$ times



Sampling Many Colorings

- Consider n simultaneous algorithm executions
- Let A denote the adjacency matrix
- Let $X_{u,i}$ be the color of vertex u in chain i
- Let $\sigma_{u,i}$ be the proposed color of vertex u in chain i
- Let $C_{u,i}$ denote if vertex u rejects its proposal in chain i
- $$C_{u,i} = \bigcup_v A_{u,v} \wedge ([\sigma_{u,i} = \sigma_{v,i}] \vee [\sigma_{u,i} = X_{v,i}] \vee [\sigma_{v,i} = X_{u,i}])$$
- Abstractly similar to matrix multiplication

The diagram illustrates an abstract matrix multiplication using shapes. On the left, a 2x2 matrix of shapes is multiplied by a 2x1 column of shapes. The result is a 2x2 matrix of shapes, where each element is the sum of the products of the corresponding row and column elements. The shapes used are: red circle, green parallelogram, orange triangle, blue diamond, pink right triangle, and purple pentagon.

$$\begin{pmatrix} \text{red circle} & \text{green parallelogram} \\ \text{orange triangle} & \text{blue diamond} \end{pmatrix} \otimes \begin{pmatrix} \text{pink right triangle} \\ \text{purple pentagon} \end{pmatrix} = \begin{pmatrix} \text{red circle} \text{ pink right triangle} + \text{green parallelogram} \text{ purple pentagon} \\ \text{orange triangle} \text{ pink right triangle} + \text{blue diamond} \text{ purple pentagon} \end{pmatrix}$$

Results

- $C_{u,i} = \bigcup_v A_{u,v} \wedge ([\sigma_{u,i} = \sigma_{v,i}] \vee [\sigma_{u,i} = X_{v,i}] \vee [\sigma_{v,i} = X_{u,i}])$
- Computing all such terms takes $O(n^{1/3})$ rounds, same as matrix multiplication for semi-rings (Censor-Hillel, Kaski, Korhonen, Lenzen, Paz, Suomela PODC '15)
- Total runtime for colorings: $\tilde{O}(\frac{n^{1/3}}{\epsilon^2})$
- For a wider class of problems (Gibbs distributions), triangle detection lower bound when $\epsilon \leq \frac{1}{32n^2}$

The diagram illustrates matrix multiplication in a semi-ring using geometric shapes. It shows three matrices in large parentheses, connected by a multiplication symbol (⊗) and an equals sign (=).
The first matrix contains four shapes: a red circle, a green parallelogram, an orange triangle, and a blue diamond.
The second matrix contains two shapes: a pink right-angled triangle and a purple pentagon.
The third matrix, representing the result, contains four terms separated by plus signs: a red circle and a pink triangle, a green parallelogram and a purple pentagon, an orange triangle and a pink triangle, and a blue diamond and a purple pentagon.

Any
Questions