Congested Clique Counting for Local Gibbs Distributions

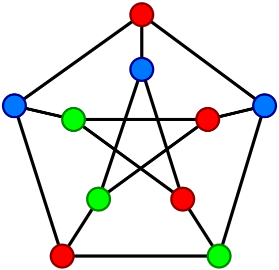
Joshua Z. Sobel

University of Iowa

October 2025

Background Information

- Goal is to estimate the size of a large set
- Estimate N by $\hat{N}$ such that $(1-\epsilon)N \leq \hat{N} \leq (1+\epsilon)N$
- For many problems can be done with $O(\frac{n}{\epsilon^2})$ samples
  Stefankovic, Vempala, Vigoda (FOCS '07)
- Improved to allow $O(\frac{n}{\epsilon^2})$ samples taken in parallel
  Liu, Yin, Zhang (arxiv '25)
- Focus on graph colorings

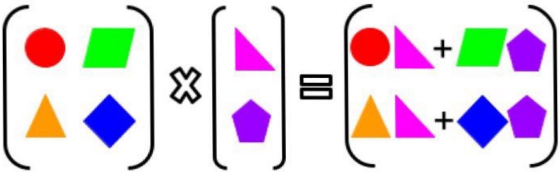


Sampling Colorings

- Markov chain from Feng, Sun, and Yin (PODC '17)
- Start from arbitrary coloring X
- Each round proceeds as follows
  - Each vertex v randomly proposes a color, σ_v
  - Each edge $\{u, v\} \in E$ passes iff
    - $\sigma_v \neq \sigma_u$
    - $\sigma_v \neq X_u$
    - $\sigma_u \neq X_v$
  - If every edge of v passes, $X_v = \sigma_v$
- Repeat $O(\log n)$ times

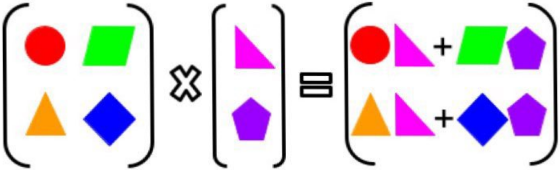
Sampling Many Colorings

- Consider n simultaneous algorithm executions
- Let A denote the adjacency matrix
- Let $X_{u,i}$ be the color of vertex u in chain i
- Let $\sigma_{u,i}$ be the proposed color of vertex u in chain i
- Let $C_{u,i}$ denote if vertex u rejects its proposal in chain i
- $\boxed{C_{u,i} = \bigcup_{v} A_{u,v} \wedge \left([\sigma_{u,i} = \sigma_{v,i}] \vee [\sigma_{u,i} = X_{v,i}] \vee [\sigma_{v,i} = X_{u,i}]\right)}$
- Abstractly similar to matrix multiplication



Results

- $C_{u,i} = \bigcup_v A_{u,v} \wedge \left([\sigma_{u,i} = \sigma_{v,i}] \vee [\sigma_{u,i} = X_{v,i}] \vee [\sigma_{v,i} = X_{u,i}] \right)$
- Computing all such terms takes $O(n^{1/3})$ rounds, same as matrix multiplication for semi-rings (Censor-Hillel, Kaski, Korhonen, Lenzen, Paz, Suomela PODC '15)
- Total runtime for colorings: $\tilde{O}(\frac{n^{1/3}}{\epsilon^2})$
- For a wider class of problems (Gibbs distributions), triangle detection lower bound when $\epsilon \leq \frac{1}{32n^2}$




Any
Questions